Coase Theory and the Coase Theorem

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Externalities

What is an externality?

A possible definition:

An action of one agent that affects the welfare of another agent, outside of the price mechanism.

For example, suppose you are in an elevator, and I walk in smoking a cigarette. Your welfare decreases due to my second-hand smoke.

This is not Pareto optimal since if you offered me $1 to refrain from smoking until I got off on my floor, I would choose to accept. I would be better off, and so would you.
Externalities

If my action affects you only through the price system, it is called a **pecuniary externality**. When actions alter prices to the betterment of detriment of agents, the equilibrium allocation changes, but the result is still Pareto Optimal in general.

Note that the definitions above are informal.

Defining externalities is a completely formal and general way is challenging, and to the best of my knowledge, has never been done.
Consider an Arrow-Debreu-McKenzie economy with many goods, many consumers and many firms. Assume all agents are price takers, transaction cost are zero, and:

Preferences are:
- Complete
- Transitive
- Continuous
- Locally Nonsatiated (or monotonic)
- Convex

Production sets are
- Bounded above
- Satisfy Free Disposal
- Closed
- Convex
Markets and General Equilibrium Theory

Existence Theorem: Competitive equilibrium exist for any initial allocation of endowments and firm ownership shares.

First Welfare Theorem: For any allocation of initial endowments and firm ownership shares, the competitive equilibrium allocations are Pareto optimal (efficient).

Second Welfare Theorem: There exists a reallocation of initial endowments and firm ownership shares such that any Pareto optimal allocation can be decentralized as a competitive equilibrium allocation.
Coase points out the following:

If a market for a valuable commodity does not open or does not exist for some reason, then any gains from trading this commodity go unrealized.

Markets may not open if transactions costs are high. Even if they do open, some beneficial trades will not be executed.

In addition, a necessary condition for a market (as envisioned by ADM) for any commodity to exist is that the ownership of the initial social endowment be completely assigned to agents, and that these ownership rights to the commodities can be fully and completely transferred to any other agent.
Coase was perfectly correct in this.

In the canonical case of air pollution, air is a valuable commodity which is a vital input into many production processes. It also has competing uses in consumption, in particular, we may wish to breath the air.

The problem is that since no one owns the air, the factory simply appropriates air until its marginal value in production is zero.

This is hardly surprising. Suppose no one owned labor. Then factories would go around and gather slaves until the marginal value of the next slave was equal to cost of catching him.

The point is that any commodity with no property rights is used past the point where social costs equal social benefits.

Coase thought markets tended to be absent in situations that resulted in externalities and so assigning property right would allow the market to find a Pareto optimal allocation for these goods as it does for more conventional goods.
So what is a Coase Theorem? Coase himself never stated one. He certainly never proved a theorem.

It is left to us to infer (meaning “pull our of the air” or “build a straw-man that describes”) what he might have had in mind.

After working on this for many years and debating the idea with many people, I have come to think of the Coase Theorem as being like love.

- We can’t agree on exactly what it is.
- Some even deny its existence.
- Others devote all their thought and energy to it, even if they don’t understand it.
- It is elusive, yet it can’t be ignored.
- Even to attempt a definition is perhaps to cheapen it.
With this caveat, let me give you my understanding of the Coase Theorem.

**Weak Coase Theorem:** If transactions costs are zero, then any allocation of property rights leads to a Pareto optimal outcome.
Coase Theorem

An illustration:

Consider a the standard case of a laundry and a steel mill.

Profit from operating steel mill: $70
Profit from operating laundry: $100
Smoke damage to laundry: $120

• We can see that no matter how property rights are assigned, the laundry ends up with them, either by buying them for a price between $70 and $100, or by refusing to sell them for less than $100.
• Even better, suppose in addition the steel mill had an avoidance technology available such as a scrubber that costs $50 to install.
• Now, if the steel mill is willing to sell air right for as low as $50, but will pay no more than $50 to acquire them. The rights end up with the laundry regardless of the initial allocation, but rather than shutdown, the steel mill employs the optimal avoidance technology.
Coase Theorem

So far, it seems the Coase theorem is true. Consider the following example:
Once upon a time, a very nice little old lady (VNLOL) lived in a very nice little old house. Her only pleasure in life was growing flowers in her garden.
One day, mean old Donald Trump (MODT) bought the lot next door. This shaded the VNLOL's garden and made it impossible to grow flowers. Fortunately, the property right to have sunshine on the VNLOL's garden existed. Unfortunately, they belonged to Donald Trump. He would be happy to sell them for $1,000,000 since this is cost of a installing a Helio-Osmotic Luminescence Exchange (HOLE) in this building. Of course, VNLOL is poor and cannot afford this. As a result, the flowers are sad.
This seems very unfair. The king decides to take this property right from MODT and give it to the VNLOL. Of course, MODT offers the VNLOL $1,000,000 to buy back these rights since this is the opportunity cost of avoiding shading the garden (That is, the cost of putting a HOLE in his building). She refuses and keeps the rights. The flowers are now happy. The end.
Coase Theorem

What is going on here? It seems it matters how property rights are assigned. Is this a counter-example to the Weak Coase theorem?

No! Both final allocations are Pareto optimal. If the VNLOL owns the property rights to sun, this is just like giving her a check for $1,000,000.

When she was poor she might have paid all she had to get the rights, but this only made them worth $1000 to her. Now that she is rich, they are worth $1,000,000 to her.

Thus, there is an income effect associated with the allocation of these valuable property rights, and so we end up a a different competitive equilibrium, but one which is nevertheless Pareto optimal (given the First Welfare Theorem).
Coase Theorem

The example does not violate the Weak Coase Theorem. It does, however, violate a stronger version of Coase Theorem:

**Strong Coase Theorem:** If transactions costs and income effects are zero, then any allocation of property rights leads to a Pareto optimal outcome.

These would not work for consumers, but if we believe that income effects are not experience by firms, then this would be the theorem to apply.
So far, the Coase Theorem is holding up.

What about the following case: Suppose that there are 200 people in an airplane going from Atlanta to Paris. For unexplained reasons, I happen to have the right to smoke up to a pack of cigarettes on this fight if I wish. No other agent is so endowed.

Assume transactions costs are zero and there are no income effects. Will we get an efficient outcome?

Note that my second-hand smoke is a **public bad**. If we follow Coase directly and consider the sort of bilateral negotiation he described, this reduces to a voluntary contribution game.
Suppose that 198 of the other passengers are damaged $1 every time I smoke a cigarette. However the 199th passenger is a rich asthmatic who suffers $10 of damage for each cigarette I smoke.

My net marginal benefit curve for smoking cigarettes is:

\[
MB(Q) = 20-Q.
\]

Being a nice smoker, I am willing to my sell my rights at exactly my marginal benefit.

The social marginal cost of my smoking is $198 + 10 = 209$ which is greater than the MB of even my first cigarette ($\text{MB}(1) = 19$). Thus, it is socially optimal for me to smoke no cigarettes at all. Any allocation where I do smoke can be Pareto dominated.
**Claim:** the only Nash equilibrium is for passenger 199 to pay me $100 and buy ten of my rights and for me to retain the remaining ten.

Why? No other agent would benefit from contributing to more smoke reduction since it only costs agents $1 to suffer the smoke, while I would require $11 to give up an additional right. Thus, all the other agents free ride on the public good provision (or public bad reduction) of passenger 199.

Thus, we seem to have done everything the Weak Coase Theorem asks for, but still, we do not find agents trading commodity to a PO allocation.

Is this a failure of the Coase Theorem? Unfortunately, yes.

But what is going on here? One might object that we did not set up a market, but instead depended on voluntary contributions to solve the issue. We looked for a Nash equilibrium rather than a “price taking” market equilibrium. However, this kind of multilateral voluntary exchange seems consistent with the bilateral exchanges that Coase uses in his paper. Maybe he had something else in mind, but it is hard to tell.
This sort of example leads to a widely held view in economics:

**The Coase Theorem is either Tautological or False**

The example above shows that without more clarity on the types of markets, institutions or mechanisms that mediate the exchange of property rights, the Coase theorem is false.

In contrast, here is a Coase Theorem that is true:

**Tautological Coase Theorem:** If agents use a transactions cost free, incentive compatible, Pareto efficient mechanism to trade property rights, then agents will trade property rights Pareto efficiently.
In short, Coase pointed out several conditions that were necessary for markets to reach a Pareto optimal outcome, but he did not give the **sufficient conditions**.

As a result, we can not get directly from Coase to a theorem that equilibrium exists and is first best. To do so, we have to address the following questions:

**A. What exactly are the commodities we should create property right for so that markets can open?**

**B. How should these markets, especially the associated price system, be structured?**
In the simple example above, the right Coasian market would have a personalized price for each agent to reduce my smoking.

Specifically, the first 198 passengers would have a Lindahl price of $1 and passenger 199 would have a price of $10. Taking these prices as given, each agent would demand anything from 0 to 20 cigarette smoking reduction.

Following Samuelson, my producer price of the public good/bad would be the sum of these: $209. At this price, I would choose to sell all the rights I have, and we get the Pareto optimal corner solution.
Coase Theory

All the essential ideas are contained in the argument above. Note that although can't quite conclude that Coase is correct (or what people say Coase meant) is correct in every circumstance, the argument does not depend on anything highly mathematical.

However, the formal statement of the problem does allow us to gain some additional insight that is obsured in the simple examples.

Here is a general ADM model with externalities:

\[
i \in \{1, \ldots, I\} \equiv \mathcal{I} \quad \text{Agents}
\]
\[
f \in \{1, \ldots, F\} \equiv \mathcal{F} \quad \text{Firms}
\]

\[N_c \quad \text{Private goods}\]
\[N_g \quad \text{Public goods}\]
\[N_d \quad \text{Directed externalities}\]
\[N_r \quad \text{Public externality rights}\]
Coase Theory

Consumers:

\[ x_i = (x_i^c, x_i^d, x_i^g, x_i^r) \in X_i \subset \mathbb{R}^{N^c+N^d+N^g+N^r} = \mathbb{R}^N \] is a consumption vector in a consumption set for agent \( i \).

\[ \omega_i = (\omega_i^c, 0, 0, \omega_i^r) \in \mathbb{R}^{N^c+N^d+N^g+N^r} = \mathbb{R}^N \] is agent \( i \)'s endowment.

\( \succ_i \) Are agent \( i \)'s preferences defined over \( X_i \subset \mathbb{R}^N \).

Note \( x_i^r \) is interpreted as rights retained by the consumers collectively. Thus, \( x_i^r \) is the level of abatement enjoyed by agent \( i \).

Assume \( \succ_i \) is

1. Complete and transitive
2. Continuous
3. Weakly convex
4. Locally non-satiated
Thus, in the model we are assuming that:

A. Only agents but not firms benefit from public goods and are damaged by externalities.

B. Only firms produce public goods and externalities.

is a production vector in a production set for firm $f$.

$$\eta_f = (0, 0, y_f^r) \in \mathbb{R}^{N + N' + N'' + N'} \equiv \mathbb{R}^N$$

is the endowment for firm $f$.

Note $y_f^r$ is interpreted as the private consumption of rights by firm $f$. 

Firms:

$$y_f = (y_f^c, y_f^d, y_f^g, y_f^r) \in Y_f \subset \mathbb{R}^{N + N' + N'' + N'} \equiv \mathbb{R}^N$$
Coase Theory

More formally, for firms we assume:

1. $Y_f$ is non-empty and closed
2. $Y_f$ is weekly convex
3. $Y = \sum_f Y_f$ is closed (since the sum of closed sets may not be closed in general.)

Feasibility:

$A$ - the set of feasible allocations is defined as follows. $a=(x_1,\ldots,x_I,y_1,\ldots,y_F)\in A$ is a feasible allocation if:

1. $\forall i \in \mathcal{I}, x_i \in X_i$
2. $\forall f \in \mathcal{F}, y_i \in Y_i$
3. $\sum x_i^c = \sum \omega_i^c + \sum y_f^c$
4. $\sum x_i^d = \sum y_f^d$
5. $\forall i \in \mathcal{I}, x_i^e = \sum y_f^e$
6. $\forall i \in \mathcal{I}, x_i^u = \sum \eta_i^u + \sum \omega_i^u + \sum y_f^u$
Prices:

\[ P \equiv \{ p = (p^c, p^d, p^g, p^n) \in \mathbb{R}^{N^c + N^d + N^g + N^r} \mid p \neq 0 \} \]

Note the personalized prices for public goods and externality rights consumed by agents.

Profit shares:

\[ 0 = (0, \ldots, 0, \ldots, 0) \in \Theta \] denotes the ownership structure of the economy, where \( 0_{i,f} \) is interpreted as the share of firm \( f \) owned by agent \( i \) and \( \forall f \in F, \sum_i \theta_{i,f} = 1 \) and \( \forall f \in F, \forall i \in I, \theta_{i,f} \geq 0 \)

Note that profits made by firms at any feasible allocation and set of prices \( (a, p) \in A \times P \) are given by:

\[ \pi_f(y_f, p) = (p^c, p^d, \sum_i p^g_i, \sum_i p^r_i)(y_f + \eta_f) \quad \text{and} \quad \pi \equiv (\pi_1, \ldots, \pi_F) \]
Coase Theory

Budget constraint:

\[ B_i(\omega_i, \theta_i, \pi, p) = \{ x_i \in X_i | \sum_i p_i \geq \sum_i x_i \} \]

Thus, note that agents pay individual prices for consuming public good and collective externality rights (abatement) but sell externality rights at the collective price.

Coasian Equilibrium:

An allocation and price vector, \((a, p) \in A \times P\) is a Coasian equilibrium relative to endowments \(\omega\) and \(\eta\) and profit shares \(\theta = (\theta_1, \ldots, \theta_i, \ldots, \theta) \in \Theta\) if and only if:

a. \( \forall i \in I, x_i \in B_i(\omega_i, \theta_i, \pi, p) \) and \( x_i \geq \bar{x}_i \forall i \in B_i(\omega_i, \theta_i, \pi, p) \)

b. \( \forall f \in F, \pi_f(y_f, p) \geq (p^c, p^d, \sum_i p_i^g, \sum_i p_i^r)(y_f + \eta_f) \forall y_f \in Y_f \).
Coase Theory

**Theorem 1:** If \((a, p)\in A \times P\) is a competitive (Coasian) equilibrium then it is Pareto optimal.

**Theorem 2:** For all \(a\in A\) such that \(a\) is Pareto optimal and \(\forall i\in I, x_i \in \text{interior}(X_i)\) there exist \(\bar{\theta} \in \Theta, p \in P\), and \((\bar{\omega}, \bar{\eta})\) where \(\sum_i \bar{\omega}_i + \sum_j \bar{\eta}_j = \sum_i \omega_i + \sum_j \eta_j\) such that \((a, p)\) is a Coasian equilibrium.

We have First and Second Welfare Theorems. And there was much rejoicing.

If externalities are directed, they are really just another commodity that is a private good or bad. Standard ADM works.

If externalities purely public goods or bads, we can simply adopt Samuelson's personalized Lindahl price system and we get existence and First and Second Welfare Theorems.
So what is the problem?

**Revelation:** How do we get agents to tell us their marginal benefit of the public good/bad/externality to give then their Lindahl prices?

**Free Riding:** Even if we knew the MB, how can we prevent the agents from demanding nothing knowing that it is unlikely we will decide to produce no public good/abatement when other agents agree to pay.

These are both standard problems in any public goods environment.

In addition, we have the following issues:
Fundamental Non-convexity

Hold the level of labor, etc. going into laundry production constant and consider what happens as you devote more and more resources to the production of steel.
Coase Theory

- If there is no externality, then laundry output is constant regardless of steel output.
- If there is even the slightest negative externality, increasing steel production decreases laundry output if input levels to laundry are held constant. One of two things must happen
  - Pollution becomes so bad that laundry out shuts down. After that point, additional steel production has no further effects, and the production frontier continues along the axis
  - If the pollution is not so bad as to shut down laundry production, the production frontier must eventually curve away from the axis.

Either way a nonconvex production set is unavoidable.
Coase Theory

Non-fundamental non-convexity

Consider a production possibility frontier between Laundry and Steel. Suppose that all the society’s resources go into these two industries. Suppose initially that the PPF is convex and that steel does not generate any externality for laundry.

Now consider what happens to the PPF as the externality gets stronger.

- If the society produces only steel (is at the corner of the PPF), then it does not matter how much smoke steel production generates. Zero laundry is produced in any event and so there is nothing to be damaged. Therefore, the PPF is unchanged where it intersects the steel axis.

- If the society produces only laundry, then it does not matter how dirty steel production might be. No steel is produce, and therefore no smoke. It follows that the PPF does not change where it intersects the laundry axis regardless of the strength of the externally.

- Finally, suppose the society produces both steel and laundry. For a given level of steel production, less laundry gets clean as the smoke becomes more severe. This means the PPF moves down, and eventually must become nonconvex.
The conclusion from this is that assuming convexity is stronger is the presence of negative externalities, but still possible.

This also causes Pigouvian taxes to fail.
What commodity should the market be for?

- Burning coal? This would get a power plant, taking prices or Pigouvian taxes as given, to set the Social marginal benefit of burning coal equal to the social marginal cost. However, it gives no incentive to install avoidance technologies, like the scrubber. The equilibrium is therefore second best.
- Actual chemical effluents, CO$_2$, NO, CO, SO$_2$, particulates, heavy metals? This is getting closer, but it is difficult to identify exactly what should be priced.
- What if I could emit particulates in the morning when the wind blows in from the sea or the afternoon when it blows out? I would want to price emissions in each period separately. What if I could take the SO$_2$ and let it go out the smoke stack, dump it into a river, into a landfill or into the ocean. Each of these has different negative effects, and so if markets are to be complete and decentralize agents' actions, they much give a price for disposing of the effluent in each of these ways (and in every other possible way).

Thus, even to write the efficient contracts we need a large and fairly arbitrary space of commodity rights, possible actions by agents, and dependence of price on time, place and state (following Debreu) for each contract. It would be very surprising if these complicated contracts could be decentralized with a linear system of prices.
Other issues:

Stock Externalities: Don't even get me started on what happens when we have stock externalities such as global CO$_2$ levels and so and have to figure out how to write the full dynamic problem correctly.

Degree off Rivalry: Finally, go back to our airplane on the way to Paris. If I sit in the front, the people in the back suffer less. The strength of the externality depends on proximity. In other words, it is not a pure public bad, but a semi-public bad, also known as a club-bad, or a local public bad. It is easy to come up with examples of externalities which are neither fully rival (private bads) nor fully non-rival (public bads).

Local public bad theory does not exist to the best of my knowledge and certainly not with the additional details that would be needed to handle externalities.

Note: These slides can be found at [jpconley.wordpress.com](http://jpconley.wordpress.com) under the Non-Technical Writings link.
Conclusions

What do economists know about the Coase's Theorem?

1. Whether it is true are not depends on how you define the Coase Theorem.

2. There are reasonable and highly policy relevant situations where it is false.

However,

3. Coase points out that the necessary conditions for the first welfare theorem may not be satisfied when externalities are present. Something economists at the time did not pay attention to.

4. It forces us to think about how best to complete the missing markets to get sufficiency and to be able to show the competitive equilibrium exists and is Pareto optimal when we complete the missing markets correctly.

Under fairly restrictive conditions, we can create externality markets that work as Coase would probably have envisioned.
Coase Theory

Broadly, we require:

• Convexity
• Avoidance technologies either do not exist or are priced correctly
• The commodity being priced or taxed is either the one that causes the externally or is perfectly correlated with externally productions
• If the externality is purely public, then personalized and aggregate prices are correctly assigned, and we have solved the revelation and free riding problem.
• If the externalities is semi-public....?

Without a doubt Coase showed us in the right road to take.

We should not blame him if we ended up in a bad neighborhood.

Instead we should simply pull our socks up and accept that sometimes problems do not have easy solutions.
Thanks!